



One-dimension mathematical model of energy parameters of a hyper concentrated mudflow

O. Natishvili, E. Kukhalashvili, G. Gavardashvili*, I. Iremashvili

Georgian Technical University, Ts. Mirtskhulava Water Management Institute, 60b, Chavchavadze Ave., Tbilisi, 0179, Georgia

Received: 10 February 2020; accepted: 20 March 2020

ABSTRACT

Natural anomalies, mudflows in particular, put many world countries before serious problems trying to develop thorough engineering solutions to combat mudflows. Mudflows are one of the dread natural calamities and relevant regulation measures are associated with the identification of their genesis, dynamics and energetic parameters. The work specifies and assesses the bed processes taking place during the mudflow movement through mudflow channels and the plans to calculate the energetic processes of currents based on the methods and scientific-technical approaches commonly accepted in hydraulics and hydraulic engineering. The problems planned in the article are solved by using classical scientific approaches and experimental study methods. The practice has evidenced that the selection of the models of the regulation measures within the action zones of mudflows needs improvement. The article, by considering the flow rheology and based on the energy equation, deduces the differential equations of uneven flow movement for different values of a bed gradient, in case of permanent and variable mudflow discharges. The obtained results allow predicting the energy parameters of the mudflow motion through non-prismatic beds by considering both, the permanent and variable value of the current discharge.

Keywords: Hyper concentrated mudflow, Rheology, Dynamics, Energetic properties, Mathematical model, Experimental study methods.

*Corresponding author: Givi Gavardashvili; E-mail address: givi_gava@yahoo.com

1. INTRODUCTION

An impact of mudflows on the environment, alongside with an ecological destruction, changes the landscape infrastructure. This is a very complex phenomenon and selecting regulatory measures in the zone of a mudflow action is quite difficult. Therefore, despite various control methods used, the efficiency of the environmental protection measures is poor and the prospects to identify adequate control measures are unfavorable. This is because mudflows are anomalous events; they deviate from the dynamic axis in motion less; influence the barriers on their way with full energy; develop the bed during their motion through the channel themselves and have high transportability. Consequently, the innovative methods to assess this phenomenon and adapted models need further improvement.

2. MATHEMATICAL MODEL OF A HYPERCONCENTRATED MUDFLOW

When mudflows are subject to a significant impact of the barriers on their way, their stability is disturbed and their motion develops in form of a wave. When the impact is insignificant, the current shifts from one stationery state into another [1-7]. Consequently, in the former case, the energetic characteristics of the current change in a step-wise manner, while in the latter case, the process takes a smooth course. Therefore, the description of this event is associated with the adaption of different models. It should be noted that a continuous or a step-wise change of such energetic characteristics, as velocity, depth and discharge are, is followed by a significant change of the amplitude of the current moving as a wave and strengthening or weakening of the impact force efficiency. Due to the complex nature of this phenomenon, a mathematical description of this

process and selection of the design parameters of the structural elements of a mudflow control structure on the basis of adapted models is very difficult due to the diversified types of mudflows occurring naturally, and the result is often catastrophic.

Following the existing statistical materials, based on the scientific works [8-13], the development of methods to identify energetic properties and adaption of the models to these phenomena is envisaged by means of analysis of easily recognizable processes first, including identifying the associations between them, and developing the relevant theory later [14-16].

In case of a variable mudflow regime, the current moves with a different shape of its free surface. This is mainly caused by the narrowing and expansion of the channel bed and changing shape of the cross section and base slope. A moving current with such a free surface when its velocity changes in the effective cross-section, is quite common. In this case, an impact of the effective cross-section on the motion must be considered. This parameter is gained by considering the width and the depth of the mudflow [17-21].

The type of the movement model of a hyper concentrated mudflow moving linearly and non-uniformly and its design model are given in Fig.

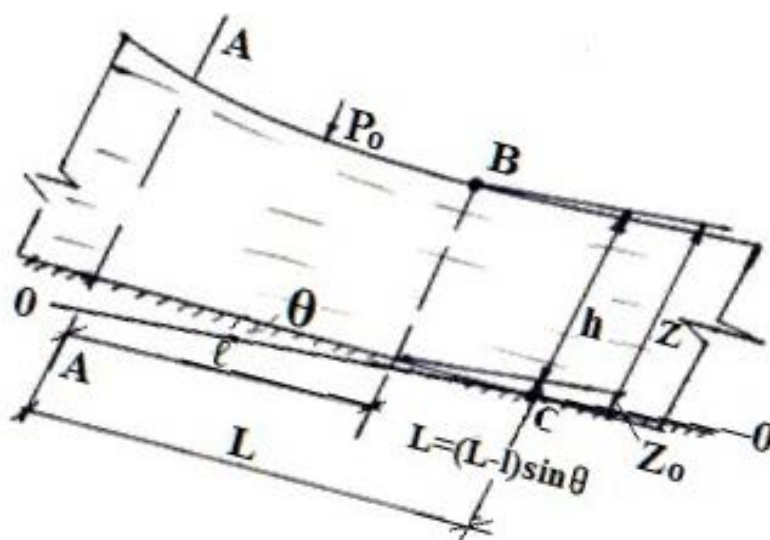


Fig. Design model of a mudflow motion

Depending on the longitudinal section of the current, when the mudflow motion is non-uniform, in order to identify the shape of the curve of the free surface, we will consider point B on its surface. The distance from the given point to the reference section A-A is l . Besides, the correlation plane is drawn across point C and it is distanced from section A-A by L . Consequently, when the distance from point B to the correlation plane O-O is Z , the value of total energy is:

$$E = Z + \frac{p}{\gamma} + \frac{\alpha V^2}{2y}. \quad (1)$$

Following the equation of energy of slowly changing motion of the current, the value of pressure loss can be presented as follows:

$$\frac{d \left(Z + P_0 / \gamma + \frac{\alpha V^2}{2g} \right)}{d\ell} = -I \quad (2)$$

Where: P_0 - is the pressure acting on the surface and equals to atmospheric pressure in case of an open bed (N/m^2); h - is the depth of the mudflow in the design section (m); ω - is the section of the effective cross-

section (m^2); Q – is mudflow discharge (m^3/sec); q – is specific mudflow discharge (m^3/sec); V – is average mudflow velocity; I – is the hydraulic gradient.

Following the equation of energy, coordinate Z of point B for the mudflow opposite to the correlation plane, is the function of rheological properties, h_0 , which is the depth equivalent to cohesiveness and angle of internal

friction $\varphi = tg\left(45^\circ - \frac{\varphi}{2}\right)$ in particular, i.e.;

$$Z = h(1 - h_0 / h)\varphi + Z_0. \quad (3)$$

When $Z_0 = (L - l)i$, by considering correction coefficient of mudflow depth K , formula (3) will be as follows:

$$Z = Kh + (L - l)i, \quad (4)$$

Consequently, the value of a piezometric gradient will be:

$$\frac{dZ}{d\ell} = K \frac{dh}{d\ell} - i, \quad (5)$$

As pressure acting on all points of the free surface of the mudflow channels $p = const$, consequently:

$$\frac{d(P/\gamma)}{d\ell} = 0 \quad (6)$$

For the 3rd member of equation (3), we will have:

$$\frac{d\left(\frac{\alpha V^2}{2g}\right)}{d\ell} = \frac{d\left(\frac{\alpha Q^2}{2g\omega^2}\right)}{d\ell} \quad (7)$$

When the current moves in the direction of motion with constant discharge:

$$\frac{d\left(\frac{\alpha Q^2}{2g\omega^2}\right)}{d\ell} = \frac{\alpha Q^2}{2g} \frac{d\left(\frac{1}{\omega_{mud}^2}\right)}{d\ell} \quad (8)$$

The association between the area and depth of the effective cross-section can be presented as follows:

$$\omega_{mud} = K\omega. \quad (9)$$

Consequently, dependence (8) will be as follows:

$$\frac{d\left(\frac{\alpha Q^2}{2g\omega^2 K^2}\right)}{d\ell} = \frac{\alpha Q^2}{2gK^2} \frac{d\left(\frac{1}{\omega^2}\right)}{d\ell} = -\frac{\alpha Q^2}{g\omega^3 K^2} \frac{d\omega}{d\ell} \quad (10)$$

I option

When the bed is not prismatic, the change of the cross section in the direction of motion will be as follows:

$$\frac{d\omega}{d\ell} = \frac{\partial \omega}{\partial \ell} + B \frac{dh}{d\ell} \quad (11)$$

By considering formula (11) in formula (7), we will gain:

$$\frac{d\left(\frac{\alpha V^2}{2g}\right)}{d\ell} = -\frac{\alpha Q^2}{gK^2\omega^3} \frac{\partial \omega}{\partial \ell} - \frac{\alpha Q^2}{gK^2\omega^3} B \frac{dh}{d\ell} \quad (12)$$

By considering formulae (12), (6) and (4) in formula (3), we will gain:

$$\frac{dh}{d\ell} = \frac{i - \frac{Q^2}{K^3 \omega^2 C^2 h} \left(1 - \frac{\alpha h K C^2}{g \omega} \frac{\partial \omega}{\partial \ell} \right)}{K - \frac{\alpha Q^2}{g K^2 \omega^3}} \quad (13)$$

If dividing the numerator and denominator in formula (13) by K and introducing denotation $i_c = i / K$, we will gain:

$$\frac{dh}{d\ell} = \frac{i_c - \frac{Q^2}{K^4 \omega^2 c^2 h} \left(1 - \frac{\alpha K h c^2}{g \omega} \frac{\partial \omega}{\partial \ell} \right)}{1 - \frac{\alpha Q^2 B}{g K^3 \omega^3}} \quad (14)$$

When $dh/d\ell = 0$ and $\frac{\partial \omega}{\partial \ell} = 0$,

$$Q = K^2 \omega c \sqrt{hi}, \quad \text{m}^3/\text{sec} \quad (15)$$

Average mudflow velocity is:

$$V = K^2 c \sqrt{hi}, \quad \text{m/sec} \quad (16)$$

When $K = 0$ and $\varphi = 1$,

$$V = C \sqrt{hi}, \quad \text{m/sec} \quad (17)$$

Formula (14), when discharge is constant, is a differential equation of smoothly changing nonuniform motion of mudflow in open non-prismatic beds and describes the regularity of the change of depth h (energetic property) along the motion.

II option

When discharge changes along the motion, equation (8) can be presented as follows:

$$\frac{d \left(\frac{\alpha Q^2}{2 g K^2 \omega^2} \right)}{d\ell} = \frac{\alpha Q}{g K^2 \omega^2} q - \frac{\alpha Q^2}{g K^2 \omega^2} B \frac{dh}{d\ell} \quad (18)$$

If considering equation (18) in (2), we will gain:

$$K \frac{dh}{d\ell} - i + \frac{\alpha Q q}{g K^2 \omega^2} - \frac{\alpha Q^2 B}{g K^2 \omega^3} \frac{dh}{d\ell} + \frac{Q^2}{K^3 \omega^2 c^2 h} = 0 \quad (19)$$

Following certain mathematical simplification and conversions in equation (19), we will have:

$$\frac{dh}{d\ell} = \frac{i_c - \frac{Q^2}{K^4 \omega^2 c^2 h} \left(1 + \frac{\alpha q K c^2 h}{g Q} \frac{dQ}{d\ell} \right)}{1 - \frac{\alpha Q^2 B}{K^3 \omega^3 g}} \quad (20)$$

When $\frac{dQ}{d\ell} = 0$ and $q = 0$

$$Q = K^2 \omega c \sqrt{hi_c} \quad \text{m}^3/\text{sec} \quad (21)$$

Equation (20) is one-dimensional differential equation of a hyper concentrated current moving through a non-prismatic bed with variable discharge.

III option

When $\frac{dQ}{dx} = q$, it can be presented as follows:

$$\frac{dh}{d\ell} = \frac{i_c - f_c - \frac{Qq}{g\omega^2 K^3}}{1 - \frac{\alpha Q^2 B}{g\omega^3 K^3}} \quad (22)$$

If presenting resistance slope of a cohesive mudflow by Shvedov-Bingham model [6-8]:

$$i_{fc} = \frac{Qv}{gh^3 bf(\beta)} \quad (23)$$

And when $\beta = h_0/h$, the value of $f(\beta)$ function in formula (23) will be:

$$f(\beta) = (1 - \beta)^2 \left(1 + \frac{1}{2}\beta \right) \quad (23)$$

The differential equation for the mudflow moving with variable discharge will be:

$$\frac{dh}{d\ell} = \frac{i_* - \frac{Qv}{gbh^3} f(\beta) - \frac{Qq}{g\omega^2 K^3}}{1 - \frac{\alpha Q^2 B}{g\omega^3 K^3}} \quad (24)$$

Equation (24) is a one-dimensional differential equation of a hyper concentrated current moving with a variable discharge through non-prismatic beds.

IV option

When the current moves with a constant speed, i.e. $q = 0$, then:

$$\frac{dh}{d\ell} = \frac{i_{fc} - \frac{Qvf(\beta)}{gbh^3}}{1 - \frac{Q^2 B}{g\omega^3 K^3}} \quad (25)$$

Equation (25) is a one-dimensional differential equation of a hyperconcentrated current moving with constant discharge through prismatic beds.

3. CONCLUSION

With the aim to solve a regulation and engineering problem of mudflows, the mathematical models were selected as an urgent means.

Differential equations, by considering various options of a hydraulic slope, were developed based on the presented models.

The differential equations were obtained based on mudflow rheology, which accurately describes the mechanism of motion.

The differential equations obtained from the energy equation allow predicting the energetic properties of a mudflow moving through non-prismatic beds – velocity and depth in case of constant or varying discharges.

The differential, equations can be used to evaluate the principal parameters determining the critical state and uniform motion of mudflows.

Acknowledgement

The paper is written under the financial support of Shota Rustaveli National Science Foundation (Grant #AR-18-1244; Project title: “Elastic mudflow-regulating barrage”)

REFERENCES

- [1] Natishvili O., Gavardashvili G., Calculation of Impact Action of a Coherent Mudflow Current in the Wave Motion Mode on a Transverse Structure, in: 9th International Scientific and Technical Conference, Modern Problems of Water Management, Environmental Protection, Architecture and Construction. Tbilisi, 2019, pp. 210-214.
- [2] Natishvili O.G., Gavardashvili G.V., Dynamics of gully-formation by considering the wave motion of flow, American Scientific Research J. for Engineering, Technology and Sciences (ASRJEST), Vol. 55, #1 (2019) 17-26. http://asrjtsjournal.org/index.php/American_Scientific_Journal/issue/view/81.
- [3] Natishvili O.G., Tevzadze V.I., Fundamentals of the Dynamics of Mudflows, Metsniereba, Tbilisi, 2007 (in Russian).
- [4] Natishvili O.G., Tevzadze V.I., One-dimensional Drift-carrying Bed Currents, Publishing House “Nauchtechizdat”, Moscow, 2012 (in Russian).
- [5] Natishvili O.G., Tevzadze V.I., Waves in Mudflows, Publishing House “Nauchtechizdat”, Moscow, 2011 (in Russian).
- [6] Natishvili O., Kruashvili I.G., Gavardashvili G.V., Inashvili I.D., Methodic Recommendations to design Anti-Mudflow Facilities (Hydraulic Calculations), Georgian National Academy of Sciences, Tbilisi, 2016 (in Georgian).
- [7] Kruashvili I.G., Kukhalashvili E.G., Inashvili I.D., Bziava K.G., Klimiashvili I.I., Mathematical model of nonuniform motion of a cohesive mudflow, J. Hydroengineering, 1-2 (17-18) (2014) 42-45 (in Georgian).
- [8] Gavardashvili G.V., Forecasting the Security of the Local People in Village Nakra of Mestia Region (Georgia) Against Floods and Mudflows, Environments, ITS, www.itspoa.com/jurnal/envi, UK, 2001, pp. 13-24.
- [9] Gavardashvili G.V., Kukhalashvili E.G., The hyper concentrated mudflow structure dynamic impact forecast, in: Proceedings of 7th International Conference Contemporary Problems of Architectures and Constructions. Florence, ITALY, 2015, pp. 397- 401.
- [10] Gavardashvili G.V., Kukhalashvili E.G., Supatashvili T., Natroshvili G., Bziava K., Qufarashvili I., The Research of Water Levels in the Zhinvali Water Reservoir and Results of Field Research on the Debris Flow Tributaries of the River Tetri Aragvi Flowing, in: 19th International Conference on Engineering and Technology, WASET, Part V, Rome, Italy, 2019, pp. 702-705.
- [11] Gavardashvili G., Kukhalashvili E., Supatashvili T., Iremashvili I., Bziava K., Natroshvili G., Qufarashvili I., The calculation of maximal and average speed of debris flow formed as a result of outstretched water wave on the land dam of Zhinvali, in: International conference on construction and Environmental Engineering, WASET, Barcelona, Spain, 2019, pp. 1029-1032.
- [12] Gavardashvili G., Kukhalashvili E., Supatashvili T., Iremashvili I., Bziava K., Natroshvili G., Qufarashvili I., Using the “CAPRA” - methodology for analysis of the critical state of the Zhinvali earth dam and risks, in: International Conference on Economic Geology and Environmental Problems. WASET, Istanbul, Turkey, 2019, pp.- 1918.
- [13] Kruashvili I.G., Kukhalashvili E.G., Inashvili I.D., Lortkipanidze D.G., Bziava K.G., Identification of the hydraulic parameters of mudflow beds, Ecological systems and Equipment, 11 (2016) 9-14.
- [14] Gavardashvili G.V., Forecasting of Erosion – Mudflow Processes in the Duruji River Basin and New Engineering and Environmental measures, Metsniereba, Tbilisi, 2003 (in Georgian).
- [15] Kukhalashvili E.G., Inashvili I.D., Bziava K.G., Kruashvili I.G., Lortkipanidze D.G., Identification of the hydraulic parameters of a cohesive mudflow with a wave-like motion through the mudflow channel. GTU, J. Hydroengineering, 1-2 (19-20) (2015) (in Georgian).

- [16] Kukhalashvili E.G., Kruashvili I.G., Khutsurauli B., Possible forms of the violation of the mudflow-forming ground stability, in: Problems of Agrarian Sciences, Collection of Scientific Works, Tbilisi, 2001 pp. 80-84.
- [17] Gregoretti C., Furlan M. & Degetto M., GIS-based cell model for simulating debris flow routing and deposition phases on a Fan, in: 5th International Conference on Debris-Flow Hazards (Mitigation, Mechanics, Prediction and Assessment). Padua, Italy, 2011, pp. 425-434.
- [18] Kailty P., Bowman E., Laue J. & Springman S., Modelling debris flow processes with a geotechnical centrifuge, in: 5th International Conference on Debris-Flow Hazards (Mitigation, Mechanics, Prediction and Assessment), Padua, Italy, 2011, pp. 339-349.
- [19] Katina R., Hsu L. & Dietrich W., On the Development of an Unsaturated Front of Debris Flow, in: 5th International Conference on Debris-Flow Hazards (Mitigation, Mechanics, Prediction and Assessment), Padua, Italy, 2011, pp. 351-358.
- [20] Kogeking A., Hubl J., Surinach E., Vilajosana I. at al., A Study of infrasonic signals of debris Flow, in: 5th International Conference on Debris-Flow Hazards (Mitigation, Mechanics, Prediction and Assessment). Padua, Italy, 2011, pp. 563-572.
- [21] Steger J.L., Worming G., Flux vector splitting of the inside geodynamic equations with application to the finite-difference methods, J. of Computer. Phys., 40 (1981) 263-293.